Roll. No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E / B. Tech (Full Time) END - SEMESTER EXAMINATIONS - NOV/DEC 2024

MA23C03 - Linear Algebra and Numerical Methods (Regulations 2023)

Time: 3 Hours | Maximum: 100 marks

Course Outcomes: The students will be able to

CO1	understand the vector spaces, linear independence, basis and its dimensions.
CO2	analyse the linear transformations and its applications through diagonalizability.
CO3	understand the orthogonality of vectors and the pupose of Gram-Schmidt process and least square approximations.
CO4	apply numerical methods to obtain exact solutions and approximate solutions to the system of linear equations.
CO5	evaluate the eigenvalues and analyze the use of eigenvalues using numerical methods.

BL - Bloom's Taxonomy Levels: L1 - Remembering; L2 - Understanding; L3 - Applying; L4 - Analysing; L5 - Evaluating; L6 - Creating.

Answer ALL questions.

PART - A (10 × 2 = 20 Marks)

Q.No	Questions	Marks	CO	BL
1.	Is the set $W = \{f(x) \in P_2(\mathbb{R}) : f'(0) = 2\}$ a subspace of $P_2(\mathbb{R})$?	2	CO1	L2
2.	Are the vectors $(3,2)$, $(-2,2)$ and $(-1,-2)$ generates $\mathbb{R}^2$? Explain.	2	CO1	L2
3.	Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, $T(1,0) = (1,4)$ and $T(1,1) = (2,5)$. What is $T(2,3)$?	2	CO2	L2
4.	Find the coordinates of the vector $(7,4)$ with respect to the basis $(2,1)$ and $(3,1)$.	2	CO2	L2
5.	State true or false with explanation. Every subset of $\mathbb{R}^2$ which contains the zero vector is orthogonal.	2	CO3	L1
6.	Let V be an inner product space. Then for all $x, y \in V$, prove that $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$.	2	CO3	L1
7.	Write any two differences between the direct methods and iterative methods for solving the system of linear equations.	2	CO4	L1
8.	Expalin the Cholesky decomposition method.	2	CO4	L1
9.	Is Jacobi rotation method applicable to all matrices? Why.	2	CO5	L1
10.	Find the singular values of the matrix $\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.	2	CO5	L2

PART - B (5 × 13 = 65 Marks)

Q.No	Questions	Marks	CO	BL
11.(a)	(i). Check whether $\{1+5x+3x^2, x+2x^2, 6x^2\}$ is a basis for $P_2(\mathbb{R})$, where $P_2(\mathbb{R})$ is the set of all polynomials of degree atmost 2.	7	CO1	L4
	(ii). Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Then show that $S \cup \{v\}$ is linearly dependent if and only if $v \in span(S)$.	6	CO1	L3
	(OR)			
(b)	(i). Is $V = \left\{ \begin{pmatrix} 2x & y \\ 3x+y & 3y \end{pmatrix} : x, y \in \mathbb{R} \right\}$ a vector space over $\mathbb{R}$ with respect to the usual matrix addition and scalar multiplication? Explain in detail.	7	CO1	L4
	(ii). Let V be a vector space and $u_1, u_2, \ldots, u_n$ be distinct vectors in V. Then prove that $\beta = \{u_1, u_2, \ldots, u_n\}$ is a basis for V if and only if each $v \in V$ can be uniquely expressed as a linear combination of vectors of β.	6	CO1	L3
12.(a)	Let $W = \{(x_1, x_2, x_3) : x_3 = x_1 + x_2\}$ be a subset of $\mathbb{R}^3$. Suppose $T : \mathbb{R}^2 \to W$ is defined by $T(x_1, x_2) = (x_1 + x_2, x_1 - 2x_2, 2x_1 - x_2)$. Determine whether T is linear and bijective transformation. Find the bases for $N(T)$ and $R(T)$. Verify the dimension theorem.	13	CO2	L5
	(OR)			
(b)	Using diagonalization, solve the system of equations $y_1' = 4y_1 + y_3$; $y_2' = -2y_1 + y_2$ $y_3' = -2y_1 + y_3$. Find the solution that satisfies the initial conditions $y_1(0) = -1$, $y_2(0) = 1$ and $y_3(0) = 0$.	13	CO2	L5
13.(a)	(i). Apply the Gram-schmidt process to transform the basis vectors $\{(1,1,1), (0,1,1), (0,0,1)\}$ with the standard inner product of $\mathbb{R}^3$ into an orthonormal basis vectors.	8	CO3	L4
	(ii). Let V is an inner product space. Then for all $x, y, z \in V$ and $c \in F$, show that $\langle x, y+z \rangle = \langle x, y \rangle + \langle x, z \rangle$ and $\langle x, cy \rangle = \bar{c} \langle x, y \rangle$.	5	CO3	L3
	(OR)			
(b)	(i). State and prove Cauchy-Schwarz inequality and triangle inequality.	8	CO3	L4
	(ii). Let W be the set of all vectors on the plane $x - 3y + z = 0$. Find the basis and dimension of the orthogonal complement of W.	5	CO3	L3
14.(a)	(i). Solve the system of linear equations $x+y+z=6$, $3x+3y+4z=20$, $2x+y+3z=13$ using Gauss elimination method with partial pivoting.	7	CO4	L4
	(ii). Use SOR method with $w = 1.25$ to solve the system of equations $4x+3y=24$, $3x+4y-z=30$, $-y+4z=-24$.	6	CO4	L4
	(OR)			

(b)	(i). Using LU decomposition method, solve the system of equations $2x_1 + x_2 + x_3 = 1$; $4x_1 + 3x_2 - x_3 = 6$; $3x_1 + 5x_2 + 3x_3 = 4$.	7	CO4	L4
	(ii). Solve the system of equations $2x + y + z = 5$, $3x + 5y + 2z = 15$, $2x + y + 4z = 8$ by Gauss - Seidal method with the initial approximations $x = 0$, $y = 0$, $z = 0$.	6	CO4	L4
15.(a)	Find the singular value decomposition of the matrix $\begin{pmatrix} 3 & 0 & -3 \\ 2 & 0 & -2 \\ 6 & 0 & -6 \end{pmatrix}$.	13	CO5	L5
	(OR)			
(b)	(i). Using Jacobi's rotation method, find all the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$	6	CO5	L4
	(ii). Find the smallest eigenvalue and the corresponding eigenvector of the matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$ using inverse power method. Let the initial approximation be $(1, 0, 0)$.	7	CO5	L5

PART - C (1 × 15 = 15 Marks)

Q.No	Questions	Marks	CO	BL
16.	(i). Let T be a linear operator on $\mathbb{R}^3$ defined by $$T(a_1, a_2, a_3) = (4a_1 + a_3, 2a_1 + 3a_2 + 2a_3, a_1 + 4a_3).$$ Determine the eigenspace corresponding to each eigenvalue and check T for diagonalizability.	8	CO2	L3
	(ii). Find the QR decomposition of the matrix $\begin{pmatrix} 1 & 2 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 2 \end{pmatrix}$.	7	CO5	L4